Recognition of Facial Expression Using Haar Wavelet Transform

M. Satiyan, M.Hariharan, R.Nagarajan

Abstract— ***This paper investigates the performance of a multiresolution technique and statistical features for facial expression recognition using Haar wavelet transform. Multiresolution was conducted up to fifth level of decomposition. Six statistical features namely variance, standard deviation, mean, power, energy and entropy were derived from the approximation coefficients for each level of decomposition. These statistical features were used as an input to the neural network for classifying 8 facial expressions. Standard deviation from the first level of decomposition was found to give better result compared to other statistical features at different level of decomposition.***

Index Terms— ***Facial expression recognition, multiresolution technique, Haar wavelet transform, artificial neural network.***

I. INTRODUCTION

Facial expressions are the most powerful and natural means of communication among human beings. An automatic recognition of facial expression through facial Action Unit (AU) has attracted much attention in the recent years due to its potential applications in behavioral science, medicine, security and human-machine interaction. The Facial Action Coding System (FACS) developed by Ekman and Friesen [1] is the most commonly used system for facial behavior analysis. Facial expression recognition is different from human emotion recognition. Facial expression recognition deals with the classification of facial muscles motion and facial feature deformation into abstract classes that are purely based on visual information, However, human emotions are a result of many different factors and their state might or might not be revealed through a number of channels such as emotional voice, pose, gestures, gaze direction and facial expressions [2]. The basic human emotions are categorized as happy, anger, sad, disgust, fear and sorrow [3]. Many researchers have proposed various methods to detect and recognize facial expression, but have still remained very challenging in real life applications.

In a previous work, Valstar et al. [4] have attempted to analyze subtle changes in facial behavior by recognizing facial muscles action units with particle filtering tracking scheme using factorized likelihoods [3] and a novel observation model that combines a rigid and a morphologic model for tracking fiducial facial points (Figure 1). Probabilistic Actively Learned Support Vector Machine (PAL-SVM) was used for AU detection. A total of 167 video clips from MMI web-based facial expression database [5] were used for testing and recognition of facial expression through 16 different AUs. To ascertain the data independency, the PAL-SVM was tested on the Cohn-Kanade database [6]. The recognition rate of the proposed method was found to be 84% when detecting 9 AUs occurring alone or in combination in input image sequences.

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M. Satiyan is a M.Sc student at the School of Mechatronic Engineering, University Malaysia Perlis (UniMAP), Perlis, Malaysia. Phone: 0127367069,
(e-mail: satiyan_316@yahoo.com)

M.Hariharan (hari@unimap.edu.my) and R.Nagarajan (nagarajan@unimap.edu.my) are lecturers at the School of Mechatronic Engineering, University Malaysia Perlis (UniMAP), Perlis, Malaysia.

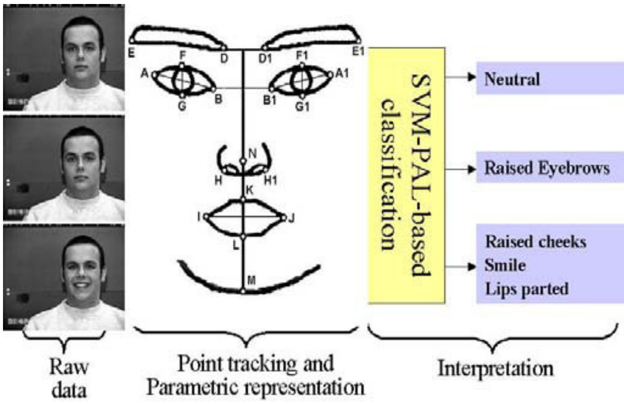


Fig. 1: The 20 fiducial facial points for tracking facial expression [2].

Peng Yang et al. [7] have utilized dynamical Haar-like features extraction method to extract the temporal information of facial AUs and expressions. They coded the extracted features into binary pattern features enthused by the binary pattern coding. At the end of the process, Adaboost was employed to classify the discriminated coded features. The proposed method has been tested on CMU facial expression database [8] and on the researcher's own facial AU database. The result showed that the proposed method has a promising performance.

Valstar et al. investigated the uses of Multilevel Motion History Images (MMHIs) and Motion History Images (MHIs) in the field of facial expression recognition [9]. Classification using MMHIs shows a lower recognition rate compared to classification using conventional MHIs after MMHIs and MHIs classified by Sparse Network of Winnows (SNoW) and a standard k-Nearest Neighbour (kNN) and tested on MMI database [5] and on the Cohn-Kanade face database [6].

The Multi-state facial component models [10] has been exploited to analyze subtle changes in facial expressions based on both permanent (mouth, eye and brow) and transient (furrows and wrinkles) facial features in a nearly frontal image sequence. Artificial Neural Network (ANN) was used to classify the extracted features from the image sequence and a recognition rate of 96.7% has been obtained through this system.

Although many researchers have used various methods to recognize facial expressions from images and videos, it is still new utilizing the multiresolution technique to process image pixel value for recognizing the facial expressions. Multiresolution methods provide powerful signal analysis tool, which are widely used in feature extraction, image compression and denoising [11]. The most popular multiresolution analysis technique is the wavelet transform. Wavelet transform can be performed for every scale and translation, resulting in Continuous Wavelet Transform (CWT). Wavelet transform can also be performed only at multiples of scale and translation intervals, resulting in Discrete Wavelet Transform (DWT). Since CWT provides redundant information and involves more computational effort, normally DWT is preferred [12]. Besides facial expression recognition, the DWT has been used in various fields such as face detection [13, 14] and emotion recognition [15, 16]. In this research, the Discrete Wavelet Transform (DWT) is utilized for its advantages. The data for feature extraction are decomposed up to fifth level of decomposition using Haar wavelet transform. Haar wavelet is one of the members of orthogonal wavelet family and it is the simplest and oldest wavelet transform. Some statistical analysis performed on the obtained wavelet coefficients. This information is given as an input to ANN. Then the optimum facial expression recognition based on classification accuracy is observed. In this research, only the subtle changes in facial behavior which represents facial expressions are analyzed by recognizing facial muscle action units and not the facial emotions.

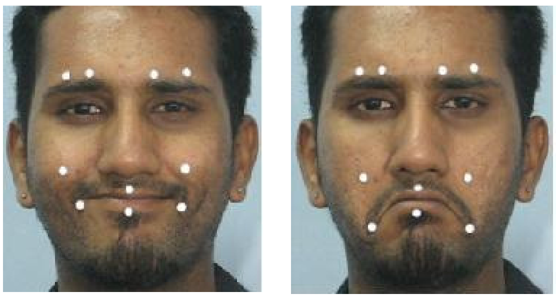
The purpose of this work is to investigate the performance of different level of wavelet decomposition which gives the highest facial expression classification accuracy and as well as to examine the statistical features which contribute to the highest result. Besides these, the performance of the learning rate of ANN in contributing classification accuracy has also been studied.

This paper is organized as follows; section 2 describes the methodology; section 3 discusses a classifier; section 4 presents result and discussion and section 5 depicts the conclusion.

II. METHODOLOGY

A. *Data Acquisition*

Acquisition of data is carried out by video recording and computer processing of facial movements. First, a set of minute luminance stickers are fixed on selected locations in a subject's face. These reflective stickers are harmless and do not impede movements of the face. A lighting system has been designed for good reflection and better video capturing. Then, the subject is asked to sit on a chair comfortably in front of a camera and remain as still as possible. Once the video recording started, the subject is instructed to perform required facial expressions (eight) such as brow raise, jaw drop, lip corner puller, brow lower, right lip corner puller, left lip corner puller, lip corner depressor and combination of brow raise and jaw drop. Figure 2 illustrates two of the facial expressions. Ten trials are taken for each movement and each trial consisted of hundred frames (800x600 pixels). Variations on face features are clearly seen through the luminance stickers which represent the muscular activity. Then the recorded videos are processed using MaxTraq (v.1.99, Qualisys, Motion Capture System) and MaxMate (v.1.3, Qualisys, Motion Capture System) [17]. MaxTraq software is mainly used for digitizing (labeling) and tracking the stickers for each frame. Figure 3 illustrates the sticker tracking responses. MaxMate software extracts the 2D coordinate values with respect to the markers for each frame as shown in Figure 4. These extracted 2D coordinate matrixes (20x100) are used for the further analysis by employing multiresolution technique to recognize eight facial expressions. A submatrix (top left (10x5)) is shown in Figure 4.



(2a) Lip corner puller (2b) Lip corner depressor

Fig. 2. Facial expression

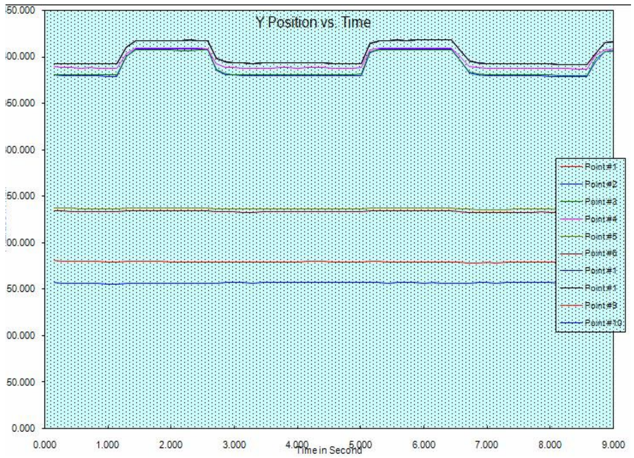


Fig.3. Sticker tracking responses

Frame #	Time	Point #1 X	Point #1 Y	Point #2 X	Point #2 Y
1	0.143	355.425	392.578	391.789	380.859
2	0.286	357.169	392.500	393.307	379.500
3	0.429	357.153	392.500	393.365	379.500
4	0.571	357.220	392.000	393.434	379.000
5	0.714	357.819	392.000	394.000	379.000
6	0.857	358.701	392.000	394.955	379.000
7	1.000	358.882	392.000	395.082	378.500
8	1.143	359.490	392.000	395.597	378.500
9	1.286	357.890	410.500	396.544	400.000
10	1.429	357.667	417.000	396.638	408.000

Fig. 4. 2D coordinate values extracted using MaxMate software.

B. *Multiresolution Technique using DWT*

Since facial movements and expression variations are dynamical in the temporal domain, it is a trend using the variations of temporal information for facial movement and expression recognition [7]. For the feature extraction, Haar wavelet (or db1 of wavelet) has been employed as a feature extraction method to capture the temporal variations of facial movement and expression from the 2D coordinate values which are time varying and non stationary. Haar wavelet is the earliest of wavelet family having lower computation expense compared with Gabor features [7]. In addition, this technique performs as a filter when decomposing the raw signals to approximation coefficients and detailed coefficients, where the high frequency band signals clustered in detailed coefficients. Since the high frequency signal is contaminated with noises, approximation coefficients are used for further analysis. The equation for the Haar mother wavelet function $\psi(t)$ is given as:

$$\psi_{a,b}(t)=a^{-1/2}\psi\left(\frac{t-b}{a}\right) \quad (1)$$

where, $a,b \in R$ $(a>0)$, R is the wavelet space. Parameter 'a' is the scaling factor and 'b' is the shifting factor.

The DWT decomposes the input signal into an approximation and detailed signal. The approximation signal is subsequently divided into new approximation and detailed signals. This process is carried out iteratively producing a set of approximation signals at different detail levels (scales) and a final gross approximation signal [11]. The filter bank structure of the wavelet transform is shown in Figure 5, where X(n) is the given input, H(n) and L(n) are the low pass and high pass filter bank coefficients, "j" represents the number of decomposition levels and A and D denote the approximation and detail spaces respectively.

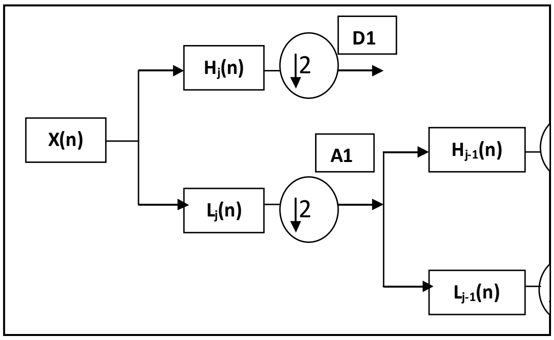


Fig. 5. Filter bank structures of DWT

In this work, the input data (2D coordinates) are decomposed up to 5th level of decomposition. The approximation coefficients from every level of decompostision are analized to observe their individual performance in facial expression recognition. The extracted wavelet coefficients provide a compact representation that shows the energy distribution of facial expression coordinates in time and frequency. In order to represent the energy distributions of wavelet coefficients more clearly, six statistical features namely variance, standard deviation, mean, power, energy and entropy are derived from the approximation coefficients for each level of decomposition. These derived statistical features from approximation coefficients of each level of decomposition (1st level to 5th level) are separately normalized between 0.1 and 0.9 and given as an input for the classifier.

III. CLASSIFICATION

An artificial neural network (ANN) is an information processing system that has been developed as a generalization of the mathematical model of human cognition [18]. ANN is one of the artificial intelligence techniques that mimic human brain. The accuracy and performance of ANN are comparable with those of human brain [19]. The ANN solves problems in classification and generalization. The basic architecture of ANN is shown in Figure 6.

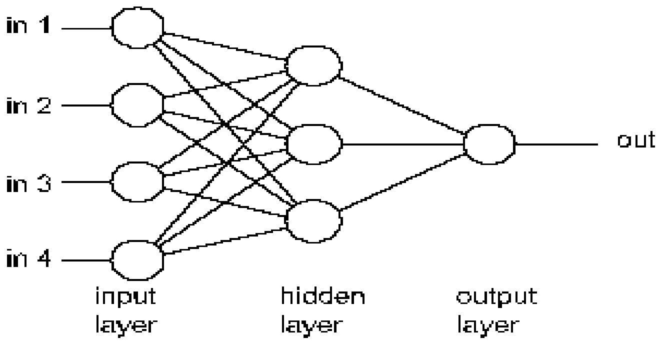


Fig. 6. Basic architecture of ANN

An ANN is also a biologically inspired classifier. Since biologically inspired classifiers are more suitable for computer based analysis of facial behavior than logically inspired methods [20], ANN is used as a classifier in this facial expression recognition system. The back propagation network (BPN) is employed as its well known multi layer perception neural network and commonly used in many applications for dealing with non-linear and complex systems [21]. The architecture of BPN used in this study is composed of four layers; one input layer, two hidden layers and one output layer. In all neurons, log-sigmoid activation functions are used. 20 neurons fixed for input layer. The number of hidden neurons is varied in step-wise to reach the maximum classification after training and testing. 10 neurons and 15 neurons are selected for the first and second hidden layer respectively. Three neurons are fixed for output layer to classify total eight facial expressions where [000] refers brow raise, [001] jaw drop, [010] lip corner puller, [011] combination of brow raise and jaw drop, [100] brow lower, [101] right lip corner puller, [110] left lip corner puller and [111] lip corner depressor. 1000 number of epoch and at momentum factor of 0.9 are selected for this neural network model. Learning Rate (LR) is varied in step-wise. The performances of BPN for different LR are presented in results section. Training is conducted until the average error fell below 0.01 or reached maximum iteration limit of 1000. Six statistical features namely variance, standard deviation, mean, power, energy and entropy are extracted from the Haar-like approximation coefficients for every level of decomposition and classified respectively to study and compare their performance. A Total of 235 samples for each feature are given as an input for the ANN to classify the facial expression accurately. Seventy percent of total samples (165 samples) are used as a training set. Remaining thirty percent (70 samples) are used as a testing set. The training and testing is carried out for 10 times in each case by reshuffling input data with the same network model. The average value of classification rate is considered as maximum classification accuracy of facial expressions.

IV. Result and Discussion

The experimental results are presented in Table 1 and Table 2. Table 1 show the results of classification accuracy for first level of wavelet decomposition to fifth level of wavelet decomposition for feature standard deviation (STD), variance (VAR) and mean. Table 2 lists the results of classification accuracy for first level of wavelet decomposition to fifth level of wavelet decomposition for feature power, energy and entropy. The LR is referring to the learning rate of ANN. The experiments are conducted with different LR in order to determine the best LR for designed ANN architecture. Each result value in every column is stated in percentage (%).The training is carried out for 10 times in each case by reshuffling input data with the same network model. So, each result in columns is the average value of 10 trials of training and testing. Among all the obtained results, it is clearly observed that the features standard deviation and variance are giving good results for the first level of decomposition compared to other features (mean, power, energy and entropy). The maximum facial expression classification accuracy is obtained as 97% for the first level of decomposition and for the feature standard deviation. This result is attained for the LR 0.001 using ANN. Classification accuracy of facial expressions has gradually decreasing from the first level to fifth level of decomposition. The fifth level of decomposition gives the lowest facial expressions classification accuracy in most of the cases. Although the results of first level of decomposition to fifth level of decomposition show a slight difference for standard deviation and variance, but the highest classification accuracy is offered by the first level of decomposition.

V. Conclusion

This paper are attempted to investigate the performance of multiresolution technique and statistical features for facial expression recognition using haar wavelet transform. Multiresolution has been conducted up to fifth level of decomposition. Six statistical features namely variance, standard deviation, mean, power, energy and entropy were derived from the approximation coefficients for each level of decomposition. These statistical features were used as an input to the neural network for classifying 8 facial expressions. Standard deviation for the first level of decomposition offers better result as 97% for the LR 0.001 compared to other statistical features and for different level of decomposition. Hence it can be concluded that the multiresolution technique using haar wavelet transform and the feature standard deviation offer promising result with an ANN classifier.

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M. Satiyan obtained his degree in mechatronic engineering from University Malaysia Perlis (UniMAP). Currently he pursuing his M. Sc in mechatronic enginerring in UniMAP. His special fields of interest include facial expression recognition, signal & image processing and Neural Network.

M. Hariharan has obtained BE in Electrical and Electronics Engineering from Bharathiar University, India and Master degree from Anna University, India. He has recently finished his Ph.D in Mechatronic Engineering at Universiti Malaysia Perlis, Malaysia and currently working as a lecturer at Universiti Malaysia Perlis, Malaysia. His research interests include speech signal processing, wavelet transform, image processing and artificial neural network. He is a member of IEEE, USA and Acoustical Society of America, USA.

Prof R Nagarajan obtained his B.E (Hons), M.Tech, and Ph.D degrees from Madras University, IIT Kanpur, and Madras Universities, India, respectively. He worked research positions in a number of universities in India, Iraq, Malaysia, the UK and Japan. He is currently with Universiti Malaysia Perlis (UniMAP), as a Professor in the School of Mechatronics Engineering. He has been working in the fields of Robotics and Intelligent Control for several years. He has received a number of research funding, research Fellowship for working abroad, research awards, medals and certificates on excellent publications and research products, and has been cited as an expert in WHO IS WHO in the world. He has visited several countries on creating research network. He has served in various committees of International Conferences around the world. Several Master, Doctoral, and Post Doctoral students have received their degrees and credentials through his supervision. He has several contributions as peer reviewed Journal and Conference papers, Books, Book Chapters, Monographs, and documented research reports. He has offered invited lectures in universities of India, Malaysia, Japan and Taiwan. His current fields of interest are in Hospital Robots, Emotion Controlled Machines, Stereo Image Processing and Robot based SLAM. Professor R Nagarajan is Life Fellow of Institute of Engineers (India). Senior Member of IEEE (USA), Member, Association of Biomedical Soft computing (BMSA), Japan, and a Member, IET (UK).

TABLE 1.
FACIAL EXPRESSION CLASSIFICATION ACCURACY FOR FEATURE STANDARD DEVIATION, VARIANCE AND MEAN.

LR	POWER					ENERGY					ENTROPY				
	1st level	2nd level	3rd level	4th level	5th level	1st level	2nd level	3rd level	4th level	5th level	1st level	2nd level	3rd level	4th level	5th level
0.0001	68.29	65.14	60.00	63.00	59.57	63.86	64.00	64.43	56.43	62.14	58.14	55.71	60.29	59.00	62.14
0.001	60.71	57.86	58.57	55.71	56.00	64.57	57.29	53.29	62.71	64.00	61.43	59.29	59.00	59.14	59.86
0.01	61.57	57.14	61.43	63.29	56.57	64.71	62.86	63.43	59.71	59.14	64.29	56.71	61.86	60.57	63.57
0.1	57.86	60.29	56.00	62.29	59.86	63.57	65.29	64.14	62.14	65.71	64.29	56.71	63.00	60.29	58.29
0.2	59.29	61.43	57.43	60.43	56.57	66.14	61.43	62.43	59.29	63.57	59.14	63.71	62.71	54.71	59.86
0.3	56.86	59.29	56.57	59.86	60.14	62.43	61.71	63.86	62.57	60.71	60.86	56.57	59.14	54.57	58.86
0.4	63.29	59.14	59.57	60.86	56.71	62.14	59.57	61.71	55.00	54.71	53.29	60.29	54.14	57.14	53.00
0.5	57.86	63.86	61.43	61.14	56.57	64.57	62.29	57.29	59.29	58.29	60.86	57.57	63.71	59.43	63.57
0.6	58.71	55.14	57.86	61.86	60.00	65.00	58.14	57.71	63.14	63.29	62.14	62.14	59.86	60.14	63.71
0.7	58.43	60.71	62.71	59.43	57.57	63.57	63.43	59.86	62.71	61.14	58.71	59.29	62.00	54.29	58.00
0.8	62.00	61.29	59.14	58.00	60.43	65.43	63.29	63.57	61.29	63.43	59.71	61.71	63.14	56.29	59.71
0.9	64.00	62.29	56.57	55.57	60.43	62.57	63.43	62.14	61.86	61.57	58.86	55.71	59.00	57.86	53.86
MAX	**68.29**	**65.14**	**62.71**	**63.29**	**60.43**	**66.14**	**65.29**	**64.43**	**63.14**	**65.71**	**64.29**	**63.71**	**63.71**	**60.57**	**63.71**

TABLE 2.
FACIAL EXPRESSION CLASSIFICATION ACCURACY FOR FEATURE POWER, ENERGY AND ENTROPY

LR	STD					VAR					MEAN				
	1st level	2nd level	3rd level	4th level	5th level	1st level	2nd level	3rd level	4th level	5th level	1st level	2nd level	3rd level	4th level	5th level
0.0001	96.71	92.57	94.29	82.43	75.57	95.00	93.86	94.71	92.71	73.00	60.86	60.71	59.86	57.14	55.71
0.001	97.00	95.36	95.14	84.14	72.57	90.71	94.43	93.86	88.14	72.71	55.57	55.71	62.14	59.71	56.57
0.01	96.00	88.14	95.29	85.14	73.00	95.00	94.43	92.14	91.71	68.29	62.86	63.43	61.29	59.00	57.57
0.1	95.71	96.29	92.86	88.00	73.86	94.43	90.43	94.14	88.43	74.00	61.86	61.57	59.86	59.86	60.71
0.2	83.57	91.43	95.71	89.57	74.29	94.57	93.29	90.57	84.86	72.29	62.14	59.43	59.86	57.29	57.71
0.3	89.71	91.43	93.29	87.57	67.14	94.43	94.57	93.71	90.29	69.29	59.57	56.00	63.57	54.71	56.71
0.4	95.86	92.71	94.29	88.57	69.86	92.00	93.86	91.71	81.43	69.00	61.14	52.00	54.57	54.29	53.71
0.5	96.14	90.14	91.00	91.29	73.00	90.29	95.14	92.71	91.57	69.14	62.43	59.00	52.57	56.71	62.43
0.6	91.29	95.86	87.29	90.00	68.14	93.00	94.57	90.14	92.29	69.57	54.29	62.14	59.14	56.86	58.86
0.7	95.57	95.86	95.43	90.43	75.29	94.00	92.71	90.00	94.00	75.14	60.14	58.57	58.14	59.71	55.86
0.8	96.00	95.86	93.14	91.14	74.86	95.86	95.00	92.29	91.29	74.00	60.00	61.57	59.29	57.71	60.86
0.9	91.71	95.14	90.71	88.57	67.71	96.71	93.57	92.29	83.57	73.43	58.00	60.00	57.29	61.14	59.86
MAX	**97.00**	**96.29**	**95.71**	**91.29**	**75.57**	**96.71**	**95.14**	**94.71**	**94.00**	**75.14**	**62.86**	**63.43**	**63.57**	**61.14**	**62.43**