Sphere Detection Analysis Performance Combined with the LDPC Decoding

Xiao Peng and Satoshi Goto

Abstract—**This paper investigates on the low density parity-check (LDPC) decoding algorithms and the detection methods of the multiple-input multiple-output (MIMO) systems. For LDPC codes, min-sum and layered decoding algorithms are discussed, and for MIMO detection, the maximum likelihood (ML) decision based on the sphere decoding algorithm is mainly analyzed. Also, the performance of the combination of the channel coding and space time coding is presented, which use the preceding methods respectively. In this LDPC coded system, the log-likelihood ratio (LLR) is propagated from LDPC decoder to the MIMO detector and then fed back, which would increase the decoding efficiency. Analysis shows that the combination certainly improves the performance of the receiver; moreover the layered LDPC decoding has better performance in this combination system.**

Index Terms— **Low density parity-check (LDPC), decoding algorithms, multiple-input multiple-output (MIMO) systems, channel coding and space time coding.**

I. INTRODUCTION

AS we all know, the fourth-generation mobile communication system (4G) has already come today and improving in the near future. In order to achieve the high spectral efficiency and high data rate, i.e. 1Gbit/s when stationary and 100Mbit/s when the client moves at high speed, some critical technologies are used in the 4G system, including multiple-input multiple-output (MIMO) and low density parity-check (LDPC) channel coding. MIMO is one of the hot technologies for fourth-generation (4G) because it can increase the capacity and link quality at no cost in frequency spectrum. Meanwhile, LDPC codes have received great attention in the channel coding area because of their excellent error correction capability and near-capacity performance. The performance which comes very close to Shannon limit (within 0.0045 dB, measured in bit error rate BER) of AWGN-channel capacity were achieved with some constructed irregular LDPC codes and very long block sizes (on the order of 10^6 to 10^7) in [1]. MIMO techniques in combination with LDPC coding has becoming a key part in almost every recent wireless standard, such as IEEE 802.11n [2] and 802.16e [3]. It is certain that the MIMO and LDPC combination will be used in an increasing number of communications environments in coming years. LDPC codes were originally proposed in 1962 by Robert Gallager [4]. Since then, the contemporary investigations in concatenated coding overshadowed LDPC codes and the hardware of that time could not support effective decoder implementations, LDPC codes remained largely unstudied for over thirty years. Until the Mackay published his work [5] in 1998, LDPC codes began to be strongly promoted. There arepotential advantages to LDPC codes which are discussed in [6]. Firstly, both abstractly and practically, LDPC codes have been proved to be capable of closely approaching the channel capacity. Secondly,

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LDPC codes have better performance than that of turbo codes which are also popular channel coding approaches in some cases, with iterative decoding algorithms which are easy to implement, and are also parallelizable in hardware. Thirdly, LDPC codes of almost any rate and block length can be created simply by specifying the shape of the parity check matrix, and the flexibility in rate is obtained only through considerable design effort. Fourthly, because the validity of a codeword is validated by its parity checks, even when errors do occur, they are almost always can detect errors especially for long codes. Lastly, on the commercial side, LDPC codes are not patent protected.

MIMO technique, on the other hand, is mainly based on the theoretical work developed by Teletar [7] and Foschini [8]. The core of MIMO is to use multiple antennas both for transmission and reception. This will increase the capacity of the wireless channel, which is expressed as the maximum achievable data rate for an arbitrarily low BER. From then, the problem has become the development of codes and schemesthat would be implemented in real systems to improve the performance. MIMO received a fillip when Tarokh et al. introduced their space-time trellis coding techniques [9] and Alamouti introduced his space-time block coding techniques to improve performance based on diversity [10]. MIMO received another boost when Bell Laboratories introduced its Bell Laboratories Layered Space-Time (BLAST) coding [11], demonstrating spectral efficiencies as high as 42 bit/s/Hz. This represents a tremendous boost in spectral efficiency compared with the current 2–3 bit/s/Hz achieved in present cellular mobile systems. This paper investigates the LDPC decoding algorithms and the detection methods of the MIMO systems. For LDPC codes, min-sum and layered decoding algorithms are discussed, and for MIMO detection, the maximum likelihood (ML) decision based on the sphere decoding algorithm is mainly analyzed. Also, the performance of the combination of the channel coding and space time coding is presented, which use the preceding methods respectively.

II. RELATED WORK

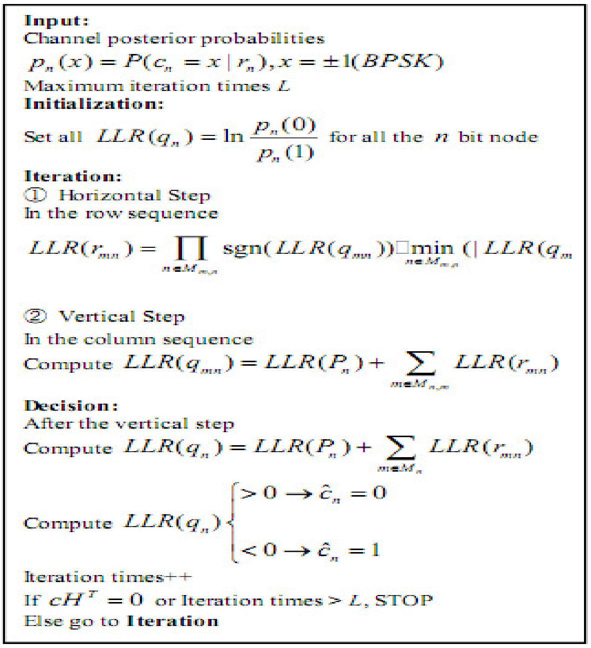
The main methods for LDPC decoding can be divided into two kinds: one is the hard-decision and the other is soft decision decoding. The former proposed by Gallager [4], is always used as an introduction example of LDPC codes. And the latter would lead to much better results. This paper will focus only this method since soft-decision decoding was discovered independently several times and as a matter of fact comes under different names. The most common ones are the belief propagation algorithm (BPA), the message passing algorithm (MPA) and the sum-product algorithm (SPA). MPA will be used in the following discussion.

MPA can also be classified into two categories: two-phase message-passing (TPMP) and turbo-decoding message-passing (TDMP) which is proposed by Mansour and Shanbhag[12]. LDPC codes are decoded iteratively using the TPMP algorithm which was proposed by Gallager [4] as well. It computes the probability values with each bit-node and the probability values with the each check-node iteratively. Each iteration consists of two phases of computations: in the horizontal phase, messages in the form of probability vectors are passed to the check nodes, where the messages are combined, and in the vertical phase, messages in the form of probability vectors are passed to the bit nodes, where the messages are combined. Updates in each phase are independent and can be parallelized. Up to now, there are many works on the TPMP algorithms and its variations, which come down are log likelihood ratio (LLR) MPA, a posterior probability (APP) MPA, min-sum MPA, normalized MPA and offset MPA. While the TDMP, which is also called as layered decoding algorithm, basically treat the parity check matrix as horizontal layers and update the extrinsic messages layer by layer. This layered decoding algorithm can improve the decoding convergence time by a factor of two and hence increases the throughput by 2X [12]. For MIMO detection, the most common method is to use minimum mean square error (MMSE) detection [13], which is thus the basis for most of the early-generation commercial systems. However, at the cost of increasing computational complexity, ML detection methods based on sphere decoding can offer significant performance advantages over MMSE, which is shown in [14] that sphere decoding outperforms MMSE by approximately 5 dB at a frame error rate (FER) of 0.1 and a greater amount at lower FERs. Thus, this paper mainly discussed the ML detection based on the sphere algorithm. For the combination of the LDPC channel decoding and MIMO detection in the receiver of wireless system, since the algorithms investigated above are all soft-input soft-output (SISO) algorithms, the soft information generated by the channel decoder should be fed back to the MIMO detector in order to increase the overall performance of the system [15]. Such iterative MIMO detection was first proposed by Hochwald and Brink [16], who described this information exchange between a soft-output sphere decoder and a turbo decoder.

III. ALGORITHMS AND PERFORMANCE ANALYSIS

A. LDPC Decoding Algorithms

The min-sum algorithm, which is widely used in many related works, especially in some hardware implementations, for example [17], was first proposed by Fossorier [18]. This algorithm can be expressed as follows:



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Input:
Channel posterior probabilities
$p_n(x) = P(c_n = x \mid r_n), x = \pm 1(BPSK)$
Maximum iteration times $L$
Initialization:
Set all $LLR(q_n) = \ln \frac{p_n(0)}{p_n(1)}$ for all the $n$ bit node
Iteration:
① Horizontal Step
In the row sequence
$LLR(r_{mn}) = \prod_{n \in M_{m,n}} \mathrm{sgn}(LLR(q_{mn}))$□$\min_{n \in M_{m,n}} (| LLR(q_m$
② Vertical Step
In the column sequence
Compute $LLR(q_{mn}) = LLR(P_n) + \sum_{m \in M_{n,m}} LLR(r_{mn})$
Decision:
After the vertical step
Compute $LLR(q_n) = LLR(P_n) + \sum_{m \in M_n} LLR(r_{mn})$
Compute $LLR(q_n) \begin{cases} > 0 \rightarrow \hat{c}_n = 0 \\ < 0 \rightarrow \hat{c}_n = 1 \end{cases}$
Iteration times++
If $cH^T = 0$ or Iteration times $> L$, STOP
Else go to Iteration
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Fig.1. Min-sum algorithm for LDPC decoding

Moreover, to some extent, the layered decoding algorithm is a simple variable of the min-sum algorithm. As described in [19], the layered decoding algorithm is always implemented with quasi-cyclic LDPC (QC-LDPC) codes.

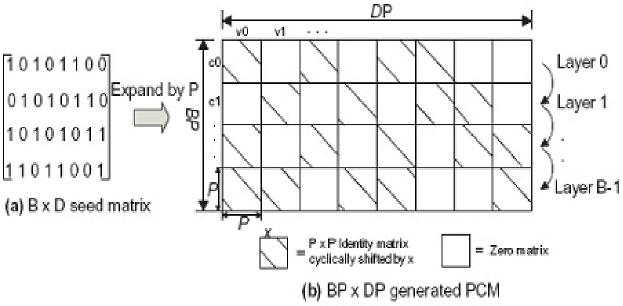

Fig.2. Structure of QC-LDPC in [19]

A QC-LDPC code as shown in Fig.2, in which the parity check matrix (PCM) is constructed from a B×D seed matrix by replacing each ’1’ in the seed matrix with a P×P cyclically shifted identity sub-matrix, where P is an expansion factor. Thus, for QC-LDPC codes, the parity check matrix can be viewed as a B row cluster × D column cluster structure by grouping variable nodes and check nodes into clusters of size P. Now let i and j denote the row cluster index and the column cluster index, a layered partially parallel decoding algorithm is given as follows:

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for iter = 0 : max iteration − 1
 for layer (row cluster) i = 0 : B − 1
  for column cluster j = 0 : D − 1
   if PCM$_{i,j}$ is a non-zero sub-matrix
   ① Read a cluster of APP data LLR($q_j$) from APP memory
   ② Read a cluster of Check data LLR($r_{mn}$) from Check memory
   ③ Calculate shift value and permute APP data
   ④ Make computation as the min-sum algorithm
   ⑤ Update new APP and Check data to memory
```

Fig.3 Layered algorithm for LDPC decoding

B. *ML Detection with Sphere Algorithm*

In a MIMO system with m transmit and n receive antennas,

the received signal vector can be written as:

$$\mathrm{y} = \mathrm{Hs} + \mathrm{n} \qquad (1)$$

where y is an n×1 complex vector whose elements are signals from receive antennas. H is an n×m complex channel matrix, in which the real and imaginary components of each element follow a Gaussian distribution with zero mean and variance 1/2. s is an n×1 complex vector whose elements are signals from transmit antennas. and s is obtained by modulating the LDPC encoded bits x. The length of x is given by nB= n×mC where mC is the number of bits per symbol. n is a complex vector of independent zero mean complex Gaussian noise entries with variance σ^2 per real component. In this paper, we assume that the channel matrix H and the noise variance σ^2 are known at the receiver.

The ML detection is an optimum receiver solution. If the data stream is temporally uncoded, the ML receiver solves the following:

$$\hat{s} = \arg\min \|\mathrm{y} - \mathrm{Hs}\|^2 \qquad (2)$$

where $\hat{s}$ is the estimated symbol vector. The ML receiver searches through all the vectors with constellation for the most probable transmitted signal vector. This implies an exhaustive investigating on s^n combinations, which is a very difficult task. Hence, it is difficult to implement directly. However, fast algorithm employing sphere decoding promoted by Babak Hassibi and Haris Vikalo [20] solved this problem. Since (2) can be factorized as:

$$\|\mathrm{y} - \mathrm{Hs}\|^2 = (\mathrm{s} - \hat{\mathrm{s}})^* \mathrm{H}^* \mathrm{H} (\mathrm{s} - \hat{\mathrm{s}}) + \|\mathrm{y}\|^2 - \|\mathrm{H}\hat{\mathrm{s}}\|^2 \qquad (3)$$

where $\hat{\mathrm{s}} = \mathrm{H}^{\dagger}\mathrm{y} = (\mathrm{H}^T\mathrm{H})^{-1}\mathrm{H}^T\mathrm{y}$.

Thus the maximum likelihood metric can be rewritten as:

$$\hat{s}_{ml} = \arg\min_{S} \|\mathrm{y} - \mathrm{Hs}\|^2 = \arg\min_{S} (\mathrm{s} - \hat{\mathrm{s}})^* \mathrm{H}^* \mathrm{H} (\mathrm{s} - \hat{\mathrm{s}}) \quad (4)$$

The principle idea of the sphere decoding algorithm is to search the closest lattice point to the received signal within a sphere radius, where each codeword is represented by a lattice point in a lattice field. That is

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$$r^2 \geq \|\mathrm{y} - \mathrm{Hs}\|^2 = (\mathrm{s} - \hat{\mathrm{s}})^* \mathrm{H}^* \mathrm{H} (\mathrm{s} - \hat{\mathrm{s}}) + \|\mathrm{y}\|^2 - \|\mathrm{H}\hat{\mathrm{s}}\|^2 \qquad (5)$$

Defining $r'^2 = r^2 - \|\mathrm{y}\|^2 + \|\mathrm{H}\hat{\mathrm{s}}\|^2$, and use Cholesky

decomposition on the matrix *H H(H H=U U, U is an upper triangular matrix), we can write (5) as:

$$r'^2 \geq (\mathrm{s} - \hat{\mathrm{s}})^* \mathrm{H}^* \mathrm{H} (\mathrm{s} - \hat{\mathrm{s}})$$
$$= (\mathrm{s} - \hat{\mathrm{s}})^* \mathrm{U}^* \mathrm{U} (\mathrm{s} - \hat{\mathrm{s}})$$

$$= \sum_{i=1}^{m} u_{i,i}^2 ((s_i - \hat{s}_i) + \sum_{j=i+1}^{m} \frac{u_{i,j}}{u_{i,i}} (s_j - \hat{s}_j))^2 \qquad (6)$$
$$= u_{m,m}^2 (s_m - \hat{s}_m)^2$$

$$+ u_{m-1,m-1}^2 (s_{m-1} - \hat{s}_{m-1} + \frac{u_{m-1,m}}{u_{m-1,m-1}} (s_m - \hat{s}_m))^2 + \cdots$$

We observe the first element:

$$u_{m,m}^2 (s_m - \overset{\square}{s}_m)^2 \le r'^2 \qquad (7)$$

And obtain

$$\left\lceil \overset{\square}{s}_m - \frac{r'}{u_{m,m}} \right\rceil \le s_m \le \left\lfloor \overset{\square}{s}_m + \frac{r'}{u_{m,m}} \right\rfloor \qquad (8)$$

Defining $r'^2_{m-1} = r'^2 - u_{m,m}^2 (s_m - \overset{\square}{s}_m)^2$, and then

$$u_{m-1,m-1}^2 (s_{m-1} - \underbrace{\hat{s}_{m-1} + \frac{u_{m-1,m}}{u_{m-1,m-1}} (s_m - \overset{\square}{s}_m)}_{\hat{s}_{m-1|m}})^2 \le r'^2_{m-1} \qquad (9)$$

Which leads to

$$\left\lceil \hat{s}_{m-1|m} - \frac{r'_{m-1}}{u_{m-1,m-1}} \right\rceil \le s_{m-1} \le \left\lfloor \hat{s}_{m-1|m} + \frac{r'_{m-1}}{u_{m-1,m-1}} \right\rfloor \qquad (10)$$

Obviously, we can continue a similar process for the bound of 2 m s □ , and so on. Therefore, this sphere algorithm avoidsthe exhaustive search of the ML detection, and reduces the complexity by recursive computation.

C. Combination of the MIMO Detection and LDPC

The SISO architecture of the combination between the LDPC channel decoding and MIMO detection in the receiver of wireless system was presented and discussed in [15].

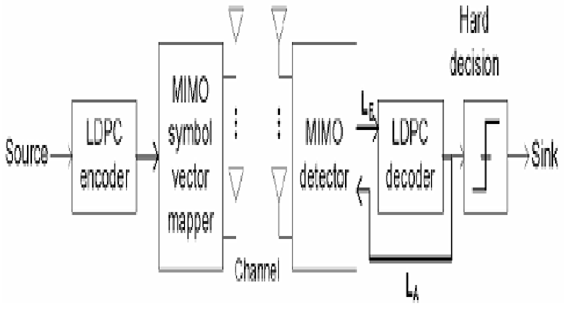

Fig.4. Block diagram of an LDPC-coded MIMO system in [15]

Fig.4 shows a simple block diagram of an LDPC-coded MIMO system. In the receiver end of this system, each receive antenna receives the sum of signals from all transmit antennas scaled by the complex channel gain and obstructed by noise.

The MIMO detector generates soft outputs from the received signal and feeds them to the LDPC decoder, which in turn provides soft data back to the MIMO detector.

During the process of the whole receiver, the LDPC decoder uses the LLR information LE which comes from the MIMO detector as the intrinsic channel information in the initialization of the iterative decoding. Since this LE is computed and obtained by a ML detection process, it is more reliable than the information just from the channel or simply setting to zero. Accordingly this advantage would improve the convergence of the LDPC decoding. And also, the LLR information LA which is fed back from the LDPC decoder to MIMO detector would be helpful for the search of the ML detection.

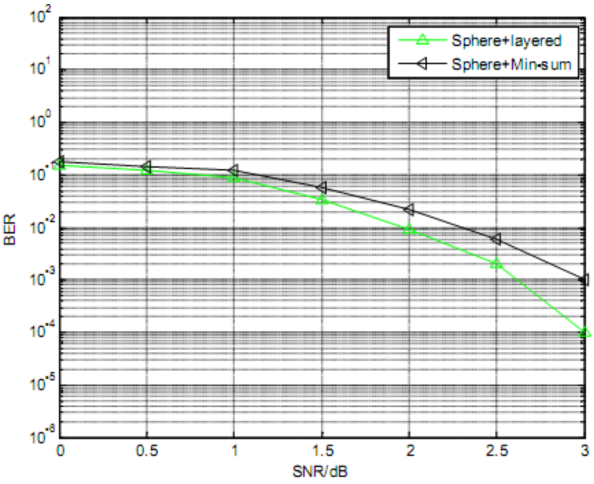

Fig.5. Simulation result of the combination

Fig.5 shows the simulation result of the combination of LDPC decoding and MIMO detection. In this simulation, the 4×4 MIMO which is modulated via 16-QAM, and (2304, 1152) QC-LDPC codes were used which is proposed by the IEEE 802.16e standard. As shown in this figure, the combination could certainly improve the performance of the receiver, and the layered LDPC decoding has better performance in this combination system.

IV. CONCLUSION

The LDPC decoding algorithms and the detection methods of the MIMO systems had been investigated; min-sum and layered decoding algorithms for LDPC codes were discussed, and for MIMO detection, the maximum likelihood (ML) decision based on the sphere decoding algorithm was analyzed. Also, the performance of the combination of the channel coding and space time coding was presented in this paper. The analysis shows that the combination would certainly improved the performance of the receiver, and the layered LDPC decoding has better performance in this combination system.

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